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Vibration Attenuation of Air Inflatable Rubber Dams with Variable Anchorage Width

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ABSTRACT

The attenuation of mechanical vibrations is a key factor in the design of structures, even more when they are related with flexible membranes as inflatable dams, where the self-excited vibrations are assumed to result from hydraulic disturbances that include unsteady water flow. In this work, the strategy for the attenuation of structural vibrations in semi-cylindrical air inflatable rubber dams proposed by Choura (1997), namely by properly varying the internal pressure, is extended to a variable anchorage width geometry adapting the mode shapes parameterization. It is demonstrated that an adequate variation in the internal pressure can modify the dynamic parameters of the structure. This approach is used to actively control the structure, measuring the induced vibration and varying the internal pressure accordingly for attenuation. The methodology followed in this work may fall under the definition of active control once that the internal pressure is changed in "real time", as a function of the self-excited vibration.

1 - INTRODUCTION

1.1 - Inflatable Rubber Dams

Inflatable dams are flexible structures attached to a foundation and have been used for various purposes: water supply, power generation, flood control or even recreation. The simplicity and flexibility of the rubber dam structure and its proven reliability are key considerations in its scope of applications. Their lengths range from a few meters to several hundred meters, but their heights are usually less than seven meters (Hsieh 1990). They are usually inflated with air, but can also be filled with water or a combination of air and water (Mysore and Liapis 1998).

2 - ON THE CAUSES OF VIBRATIONS

Inflatable rubbers dams are flexible structures and due to their low stiffness they change their geometry accordingly to the pressure distribution along the surface. The occurrence of vibrations can be very complex in nature once that it's a fluid/structure interaction. Nevertheless, we can distinguish the following types of vibrations (Gebhardt, M., et al. 2010): vibrations of the nappe, vibration due to pressure fluctuations and vibrations due to uplift forces.

2.1 - Vibrations of the nappe

These vibrations arise in inflatable dams with deflectors and can be observed at small overflow depths and low tailwater levels. Due to the deflector, an air cavity is formed between the membrane and the nappe (Fig. 1). If this cavity is not ventilated free-surface undulations might occur but result only in small deformations.

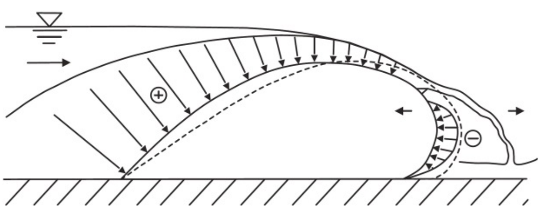

Fig. 1 – Vibrations of the nappe.

2.2 - Vibrations due to pressure fluctuations

These vibrations are caused by an unstable nappe separation on the downstream face of the rubber dam (Fig. 2). The consequent pressure fluctuations lead to vibrations and large deformations.

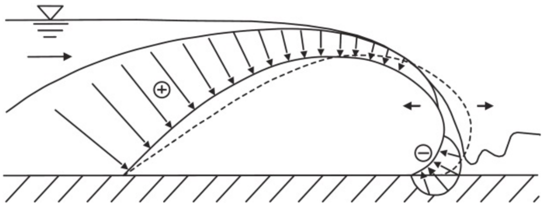

Fig. 2 – Vibrations due to pressure fluctuations (Gebhardt, M., et al. 2010).

2.3 - Vibrations due to uplift forces

These vibrations occur during high overflow and increased downstream water level. The presence of high gradient pressures leads to a change in the cross section to an aerodynamic shape like a wing. Due to the contraction of the flow a negative pressure area is formed on the upstream face of the rubber body which lifts the membrane and leads to a change of the flow field (Fig. 3). With the increased flow resistance the membrane is pushed down again.

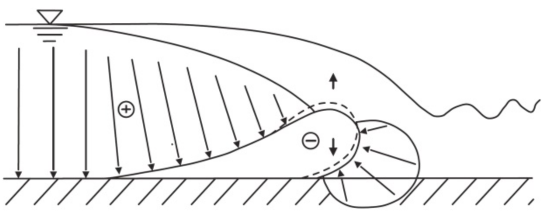

Fig. 3 – Vibrations due to uplift forces. (Gebhardt, M., et al. 2010).

All of these situations share an interesting common trait that is the self-excited nature of the vibration. The vibration of the rubber dam feeds the excitations, i.e. water flow fluctuations and/or pressure gradients, which makes the problem very challenging. There are some solutions pointed by the researchers to break the cycle excitation/vibration namely, splitters or breakers on the membrane body to break the nappe and avoiding long air cavities and pressure fluctuations or ventilation of the air cavity to eliminate the low pressures, to name just a few. In this work, an approach to change the dynamic properties of the rubber dam is followed measuring the induced vibration and varying the internal air pressure accordingly for attenuation. This methodology is appropriate to decrease the problems of cost and construction of splitters or air ventilation and may fall under the definition of active control once that the internal pressure is changed in "real time", as a function of the self-excited vibration.

3- MECHANICAL MODEL

3.1 - Equation of Motion

The dynamics of an air-inflated cylindrical membrane dam (Fig. 4) can be described by the following equations (Choura 1997):

$$\frac{\partial^4 w}{\partial \theta^4} + \frac{\partial^2}{\partial \theta^2}\left(w - \frac{\mu R}{P}\frac{\partial^2 w}{\partial t^2} \right) + \frac{\mu R}{P}\frac{\partial^2 w}{\partial t^2} = 0$$

$$w = \frac{\partial v}{\partial \theta} \tag{1}$$

with the following boundary conditions for $w(\theta, t)$ and $v(\theta, t)$, radial and tangential displacements at any time t, respectively:

$$w(0,t) = w(\alpha,t) = v(0,t) = v(\alpha,t) = 0 \tag{2}$$

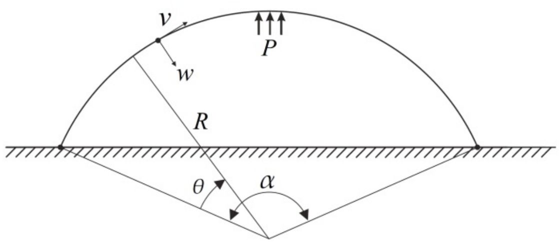

Fig. 4 – Geometry of an inflatable dam.

where R is the radius of the circular rubber dam model, μ is the mass per unit length and α is the central angle of the circular membrane arc. The dynamics of the air-inflated membrane is based on the assumptions that the deflections are small, extension and the effect of damping are negligible during deformation and the membrane is long, which allows a two-dimensional model.

3.2 - Controlled Pressure

It is assumed that the internal pressure $P(t)$ is composed by a nominal pressure P_0 and a controlled pressure $P^*(t)$:

$$P(t) = P_0 + P^*(t) \tag{3}$$

The main objective of this methodology is assure a minimum of internal pressure for sustain the equilibrium configuration. Moreover, the interest is to set a law for the variation of $P^*(t)$ that attenuates the vibration of the inflatable structure, once that the dynamic properties depend upon the internal pressure. The radial displacement $w(\theta,t)$ can be written separately, in space and time, as:

$$w(\theta,t) = \sum_{n=1}^{\infty} W_n(\theta) T_n(t) \tag{4}$$

where $W_n(t)$ are the eigenfuctions from the problem described in Eq. 1 with constant pressure and $T_n(t)$ are the modal amplitudes. Hsieh (1990) shown that the eigenvalue problem with a constant pressure leads to the following characteristic equation:

$$2(1-\cos a_n \cosh b_n) + \frac{b_n^2 - a_n^2}{a_n b_n} \sin a_n \sinh b_n = 0 \tag{5}$$

where:

$$\begin{aligned} a_n &= \frac{\alpha}{\sqrt{2}} \sqrt{1+\lambda_n + \sqrt{1+6\lambda_n + \lambda_n^2}} \\ b_n &= \frac{\alpha}{\sqrt{2}} \sqrt{-1-\lambda_n + \sqrt{1+6\lambda_n + \lambda_n^2}} \\ \lambda_n &= \frac{\mu \omega_n^2 R}{P_0} \end{aligned} \tag{6}$$

and the eigenfunctions that correspond to both radial and tangential displacements are:

$$\begin{aligned} W_n(\theta) = A_n \Big(& \sin a_n \frac{\theta}{\alpha} + H_n \cos a_n \frac{\theta}{\alpha} \\ & + \frac{a_n}{b_n} \sinh b_n \frac{\theta}{\alpha} - H_n \cosh b_n \frac{\theta}{\alpha} \Big) \\ V_n(\theta) = A_n \frac{\alpha}{a_n} \Big(& -\cos a_n \frac{\theta}{\alpha} + H_n \sin a_n \frac{\theta}{\alpha} \\ & + \cosh b_n \frac{\theta}{\alpha} - H_n \frac{a_n}{b_n} \sinh b_n \frac{\theta}{\alpha} \Big) \end{aligned} \tag{7}$$

where A_n is a constant and H_n is given by:

$$H_n = \frac{\sin a_n + \dfrac{b_n}{a_n} \sinh b_n}{\cos a_n - \cosh b_n} \tag{8}$$

and W_n and V_n satisfy:

$$\begin{aligned} \frac{\mathrm{d}^4 W_n}{\mathrm{d}\theta^4} + (1+\lambda_n) \frac{\mathrm{d}^2 W_n}{\mathrm{d}\theta^2} - \lambda_n W_n = 0 \\ W = \frac{\mathrm{d}v}{\mathrm{d}\theta} \end{aligned} \tag{9}$$

with the boundary conditions defined in Eq. (2). With the separation of variables method defined in Eq. (4), the differential Eq. (1) can be re-defined as:

$$\frac{\dfrac{\mathrm{d}^4 W_n}{\mathrm{d}\theta^4} + \dfrac{\mathrm{d}^2 W_n}{\mathrm{d}\theta^2}}{\dfrac{\mu R}{P_0}\left(W_n - \dfrac{\mathrm{d}^2 W_n}{\mathrm{d}\theta^2} \right)} = \omega_n^2 = -\frac{\dfrac{\mathrm{d}^2 T_n}{\mathrm{d}t^2}}{\left(1 + \dfrac{P^*}{P_0}\right) T_n} \tag{10}$$

which allows to write the modal amplitudes, in a matrix form, as:

$$\frac{\mathrm{d}^2 \mathbf{T}}{\mathrm{d}t^2} + \left(1 + \frac{P^*}{P_0}\right) \mathbf{\Omega T} = 0 \tag{11}$$

where the vector $\mathbf{T} = [T_1, T_2, T_3, \cdots]^{\mathrm{T}}$ and the matrix $\mathbf{\Omega} = \mathrm{diag}\left(\omega_1^2, \omega_2^2, \omega_3^2, \cdots\right)$.

To control the vibrations of the membrane we need to guarantee that the system of Eq. (11) is asymptotically stable. The most common method to analyze the stability of such systems is the Lyapunov method. In this approach, a called Lyapunov function, denoted here by $E(\mathbf{T})$, is a real scalar function of the vector $\mathbf{T}$, which has continuous first partial derivative and satisfies the following conditions:

1. $E(\mathbf{T}) > 0$ for all values of $\mathbf{T}(t) \neq 0$
2. $\dfrac{\mathrm{d}E(\mathbf{T})}{\mathrm{d}t} \leq 0$ for all values of $\mathbf{T}(t) \neq 0$

With the previous conditions it can be stated that if there exists a Lyapunov function for a given system, that system is stable. In addition, if the function $\dot{E}(\mathbf{T})$ is strictly less than zero, the system is asymptotically stable. In mechanical systems the sum of kinetic and potential elastic energy is a good candidate for the Lyapunov function. Taking into account Eq. (11) a Lyapunov function can be defined as:

$$E(\mathbf{T}) = \frac{1}{2}\left[\left(\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t}\right)^{\mathrm{T}}\left(\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t}\right) + \mathbf{T}^{\mathrm{T}}\mathbf{\Omega}\mathbf{T}\right] \tag{12}$$

Writing the time derivative of the Lyapunov function and taken into account Eq. (11):

$$\begin{aligned}\frac{\mathrm{d}E(\mathbf{T})}{\mathrm{d}t} &= \left(\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t}\right)^{\mathrm{T}}\left(\frac{\mathrm{d}^2\mathbf{T}}{\mathrm{d}t^2} + \mathbf{\Omega}\mathbf{T}\right) \\ &= -\frac{P^*}{P_0}\left(\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t}\right)^{\mathrm{T}} \mathbf{\Omega}\mathbf{T}\end{aligned} \tag{13}$$

For the time rate of $E(\mathbf{T})$ to be negative, i.e. the energy of the system is decreasing and the system is stable or asymptotically stable, one can select P^* as:

$$\begin{aligned}P^* &= kP_0\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right)^{\mathrm{T}} \mathbf{\Omega}T \\ &= kP_0\sum_{i=1}^{\infty}\omega_i^2\frac{\mathrm{d}T_i}{\mathrm{d}t}T_i \text{ with k>0}\end{aligned} \tag{14}$$

Thus, we have a law for the control pressure that is a non-linear function of the modal displacements and velocities. The constant k must be properly tuned for the desired rate of convergence to equilibrium. In practice, this approach is achieved with a Proportional and Derivative (PD) controller.

4- CASE STUDY

4.1 - Inflatable Rubber Dam

Let us consider an inflatable dam with the parameters presented in Table 1.

Table 1 – Properties of the inflatable dam.

R (m)	α (rad)	θ (rad)	μ (Kg/m)	P_0 (N/m)
2,5	1,5 π	$\pi/4$	1,0991	5×10^4

The first four mode shapes are depicted below:

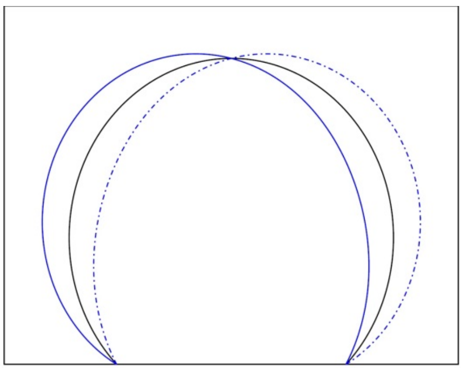

Fig. 5 – 1st Mode Shape.

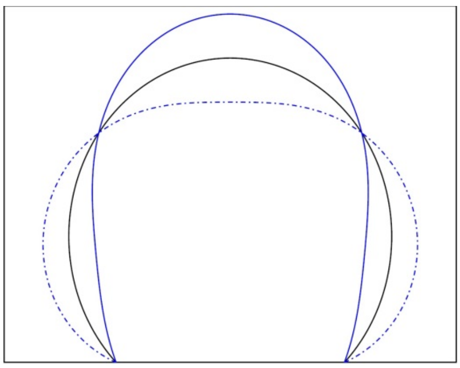

Fig. 6 – 2nd Mode Shape.

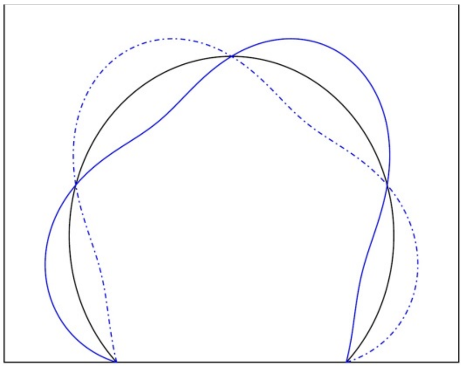

Fig. 7 – 3rd Mode Shape.

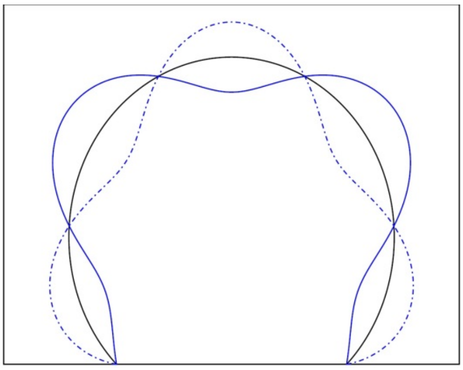

Fig. 8 – 4th Mode Shape.

Analyzing the mode shapes represented in Fig. 5-8 it can be seen that the first mode is anti-symmetric with one node, the second is symmetric with two nodes and so on. It is interesting to note that in the odd mode shapes one can distinguish that vibration is predominant in horizontal direction, in contrast with the even mode shapes where the vibration is predominant in vertical direction. This is important if one consider the causes of vibration where were identified that vibrations of the nappe and due to pressure fluctuations induce vibrations in horizontal direction and vibrations due to uplift forces induce vibrations in vertical direction. Identify the causes of vibration can be helpful to determine which are the predominant modes that are excited.

As one can see in Eq. (6) the natural frequencies are a non-linear function of the internal air pressure. In Fig. 9 the influence of the internal pressure on the natural frequencies for the first four modes is represented and, as expected, the natural frequencies increase with the internal air pressure once that the stiffness of the membrane is increased with the pressurization. These results should emphasize the available possibilities in these systems to control them, namely by properly varying the internal air pressure. For the inflatable rubber dam considered in this case study the associated first four natural frequencies are $\omega_1 = 72,29$ rad/s , $\omega_2 = 181,17$ rad/s , $\omega_3 = 292,02$ rad/s and $\omega_4 = 393,34$ rad/s .

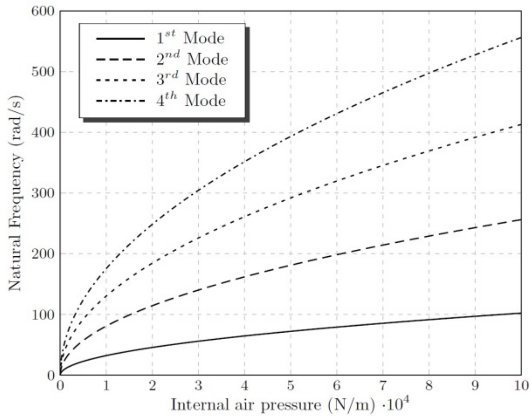

Fig. 9 – Influence of internal air pressure in the natural frequencies.

In Fig. 10 the time response of the uncontrolled membrane dam is presented where only the first mode was initially excited with $T_1(0) = 0,05$. The time response of the controlled membrane is depicted in Fig. 11 and the respective controlled air pressure is represented in Fig. 12.

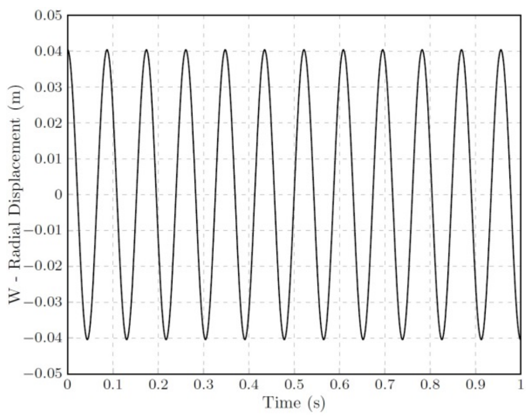

Fig. 10 – Uncontrolled time response of the radial displacement.

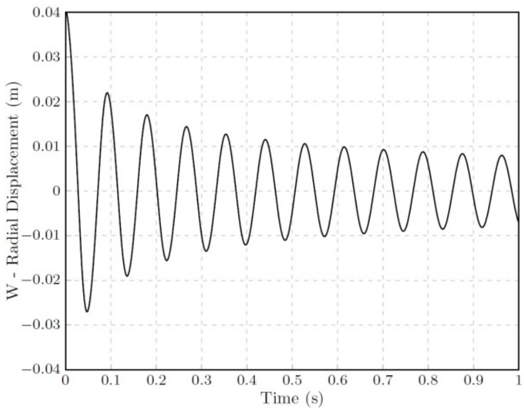

Fig. 11 – Controlled time response of the radial displacement.

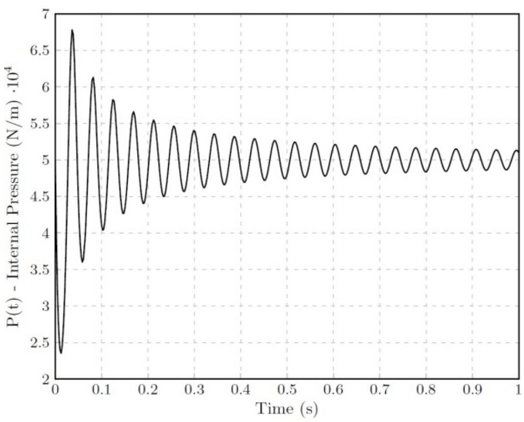

Fig. 12 – Controlled internal air pressure.

The constant k was defined as:

$$k = \frac{\sqrt{2}}{\omega_1^3 T_1^2(0)} \tag{12}$$

With this constant one can define the gain of the system and tune it as desired. If faster time decays are needed the constant can be increased but care must be taken, namely for overshooting problems that can put in risk the structural integrity of the rubber dam membrane. From Fig. 11 it can be viewed that the damping ratio varies in time, being higher at the beginning and low afterwards. This is due to its dependency with the modal amplitudes. The controlled air pressure represented in Fig. 12 follows the same behavior in turn of the nominal pressure $(5\times10^4\ \text{N/m})$ but with double the frequency. In fact, this occurs because the variable pressure $P^*(t)$ is sensible to the sign of the modal amplitude $T_i(t)$ but also to the sign of its time derivative $\mathrm{d}T_i / \mathrm{d}t$, as one can see in Eq. (14). In Fig. 13 the time response of the uncontrolled membrane dam is presented where the first three modes were initially excited with $T_1(0) = 0,05$, $T_2(0) = 0,01$ and $T_3(0) = 0,005$. The time response of the controlled membrane is depicted in Fig. 14 and the respective controlled air pressure is represented in Fig. 15. In this second example the presence of different vibration modes can be seen. Nevertheless, the vibration is attenuated as one can see in Fig. 14, where the controlled radial displacement follows the same tendency of the first example where only one mode was considered. The presence of different vibration modes leads to an irregular controlled air pressure (Fig. 15) however the general characteristics are the same of the first example (Fig.12).

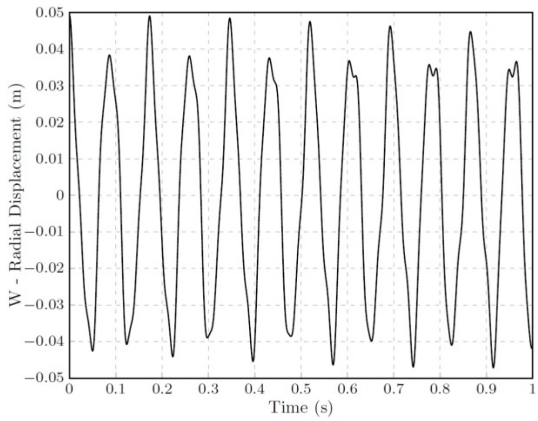

Fig. 13 – Uncontrolled time response of the radial displacement.

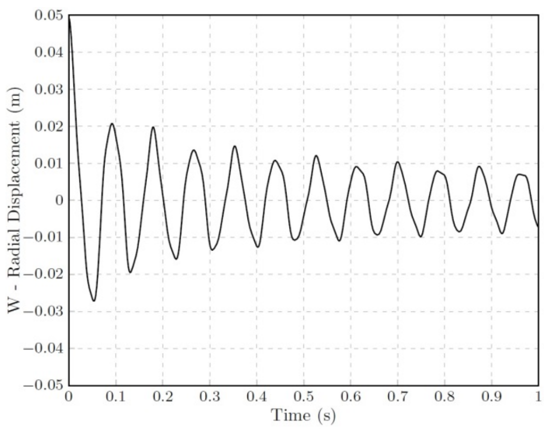

Fig. 14 – Controlled time response of the radial displacement.

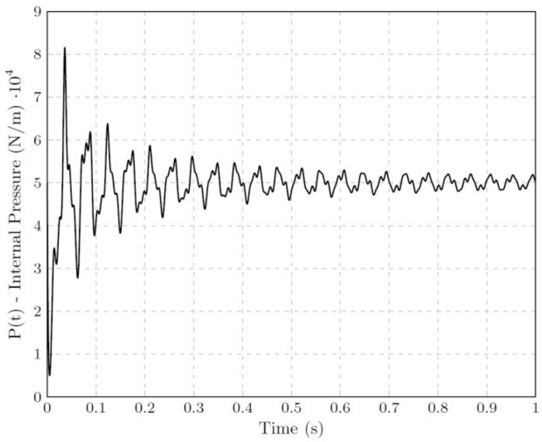

Fig. 15 – Controlled internal air pressure.

5- DISCUSSION

Analyzing Fig. 11 and Fig. 14 it is clear that this methodology is capable of attenuate the vibrations of the rubber dam membrane. This approach is used to actively control the structure, measuring the modal amplitudes and varying the internal pressure as desired. The methodology followed here may fall under the definition of active control once that the internal pressure is changed in "real time", as a function of the modal amplitudes. It is worth to mention that the damping ratio varies in time. In fact, at the beginning the damping is high and low afterwards. This is due to the dependency of the damping ratio on the modal amplitudes. It should be noted that the displacement tends to a limit that is different from 0, i.e. the vibration is attenuated and not eliminated once that the energy is "controlled" and not entirely dissipated. It is demonstrated that a proper variation in the internal pressure can change the dynamic parameters of the structure. This methodology should enlighten to the advantage of varying the internal pressure for vibration attenuation.

6- CONCLUSION

A strategy for vibration attenuation of air inflatable rubber dams was followed in this work. It was demonstrated that the self-excited vibration can be actively controlled measuring the induced vibration and change the internal air pressure accordingly for attenuation. Moreover, the strategy followed in this work can be applied in different systems with variable stiffness and should emphasize the advantages of these systems (inflatable structures but not limited to) where the dynamic properties can be controlled as desired.

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